Absence of Barren Plateau in Quantum Convolutional Neural Networks

arXiv:2011.02966

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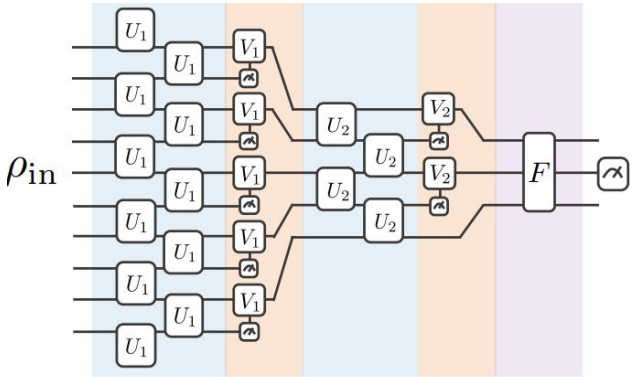
Is there a barren plateau problem in the quantum CNN?



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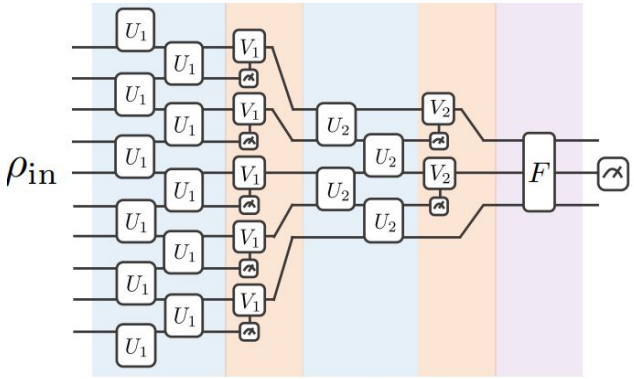
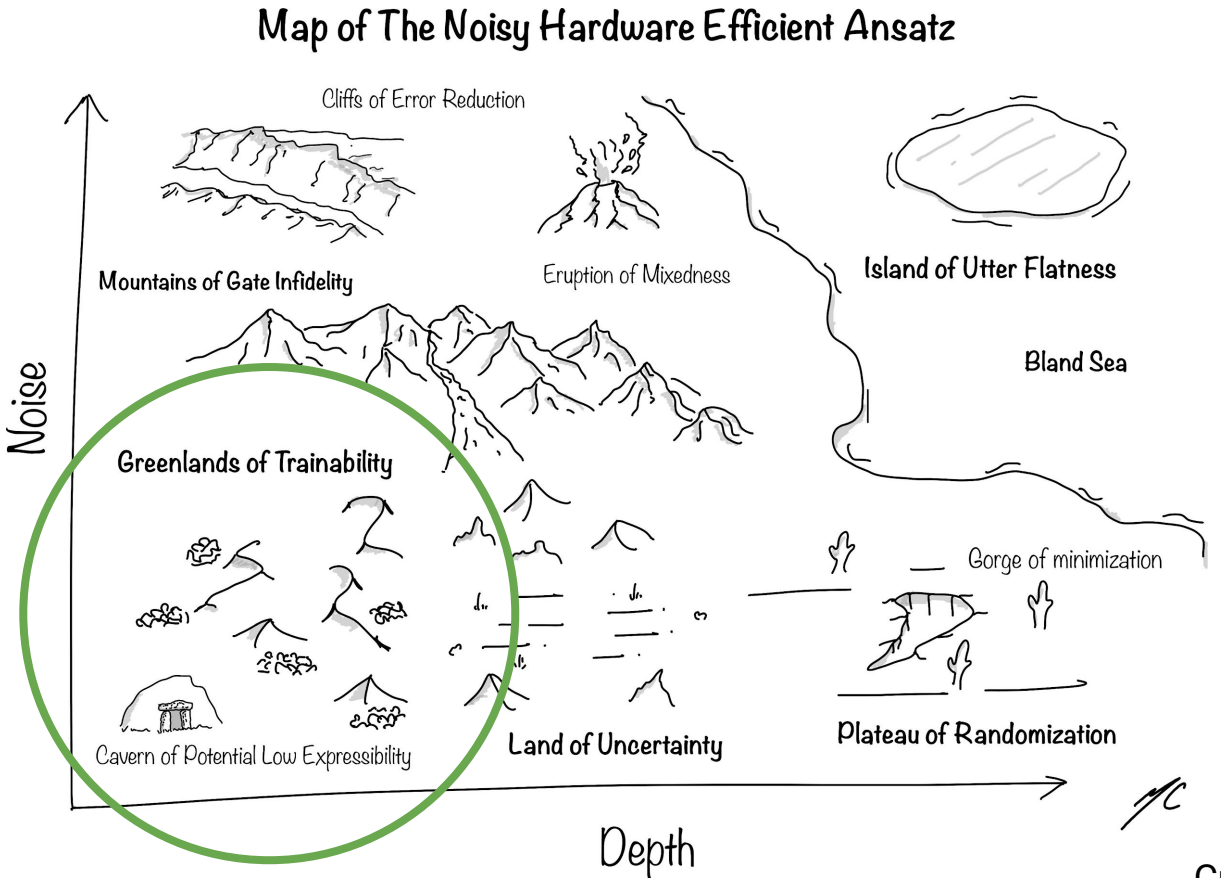


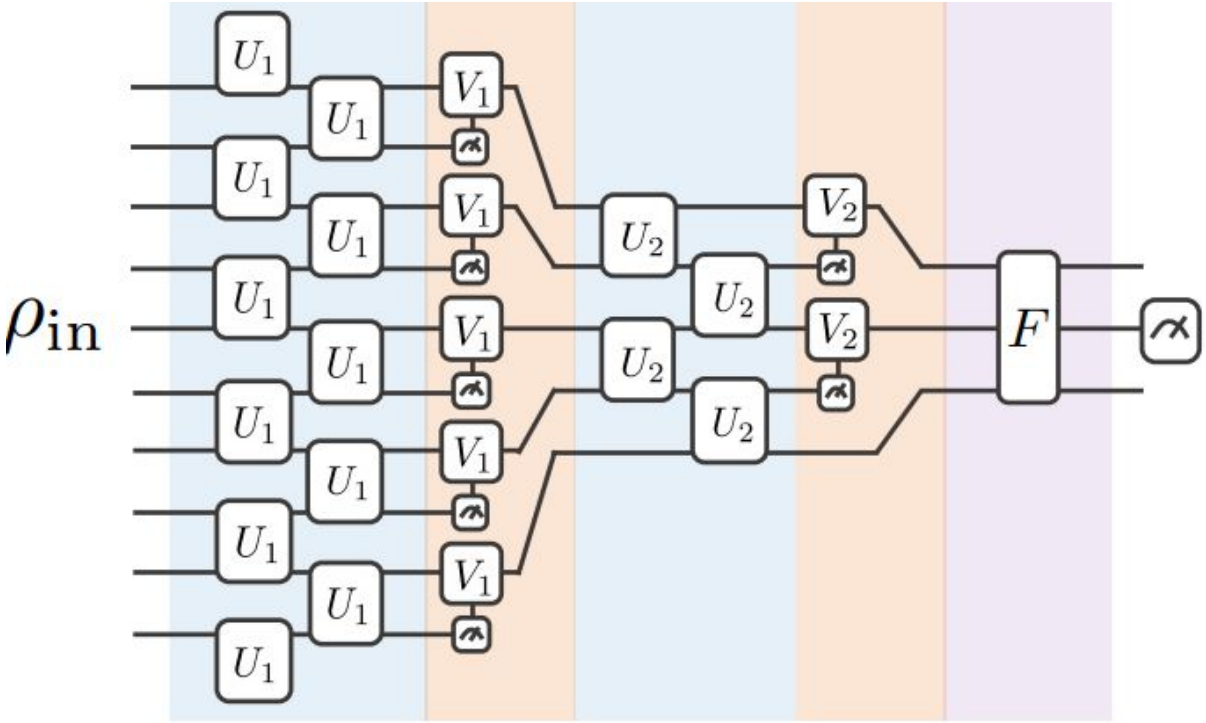
Our work

Is there a barren plateau problem in the quantum CNN?



Credit: Marco Cerezo

Quantum Convolutional Neural Network (QCNN)



Cong et al, *Quantum Convolutional Neural Network*, 2018, arXiv:1810.03787

Grant et al., *Hierarchical quantum classifiers*, 2018, arXiv:1804.03680

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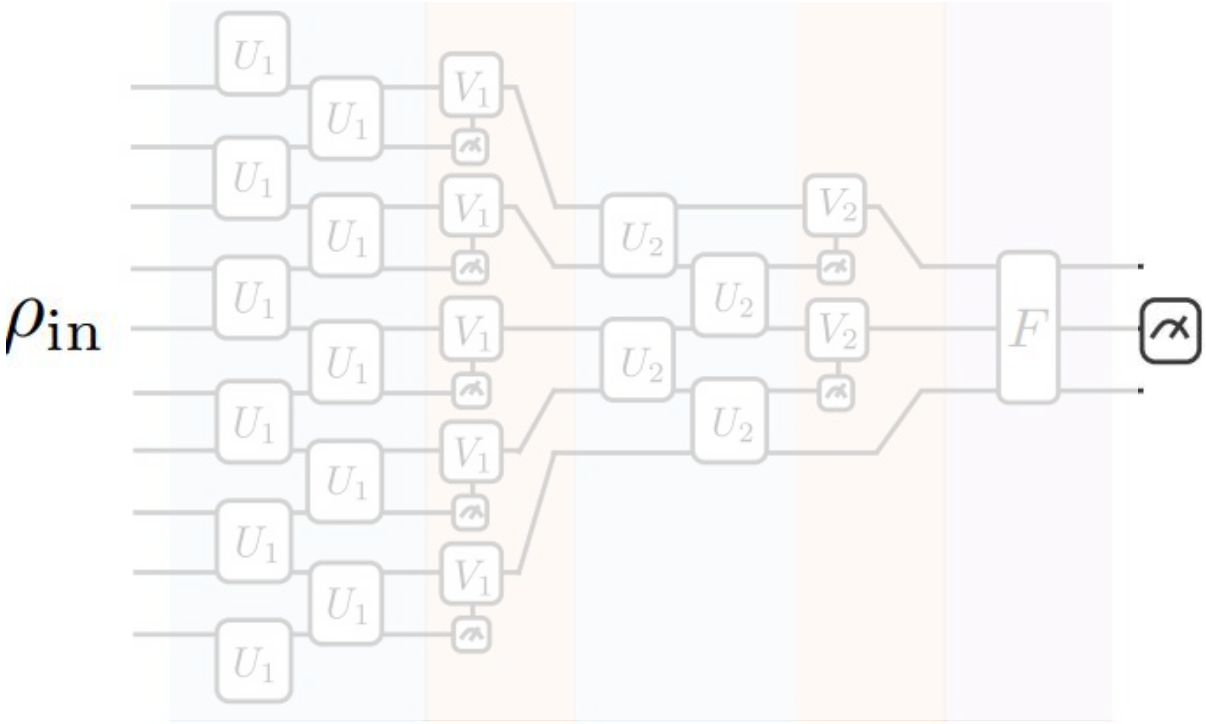

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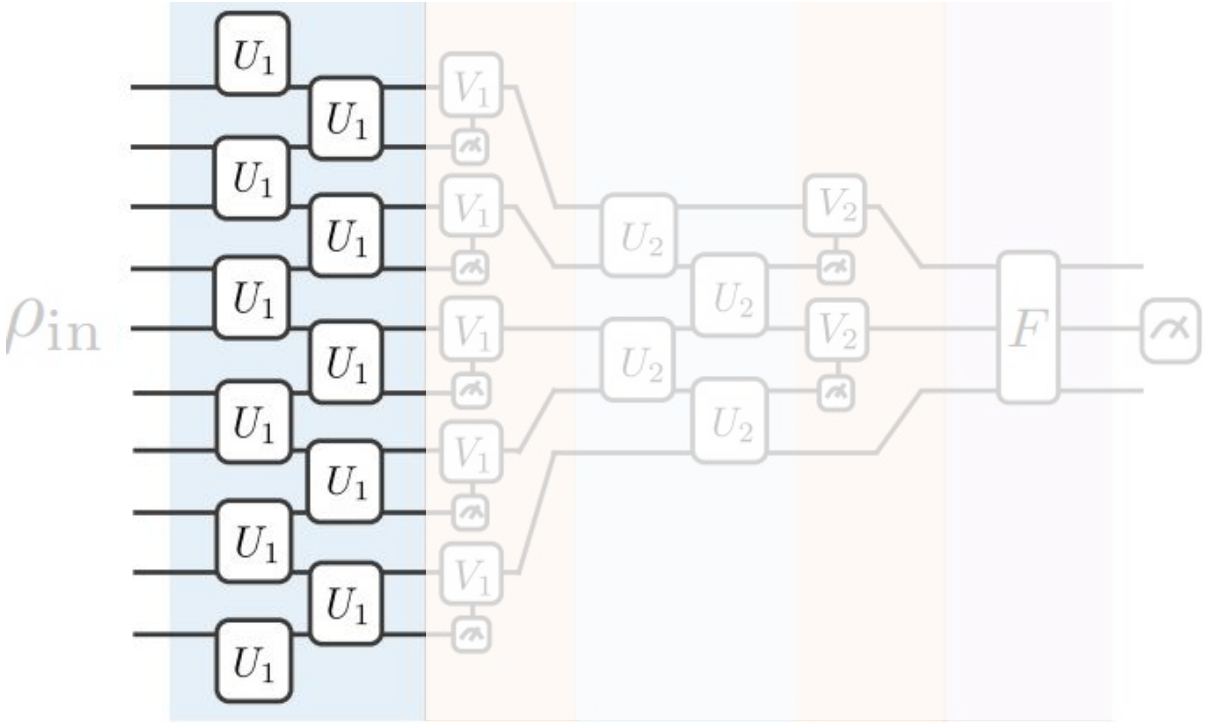

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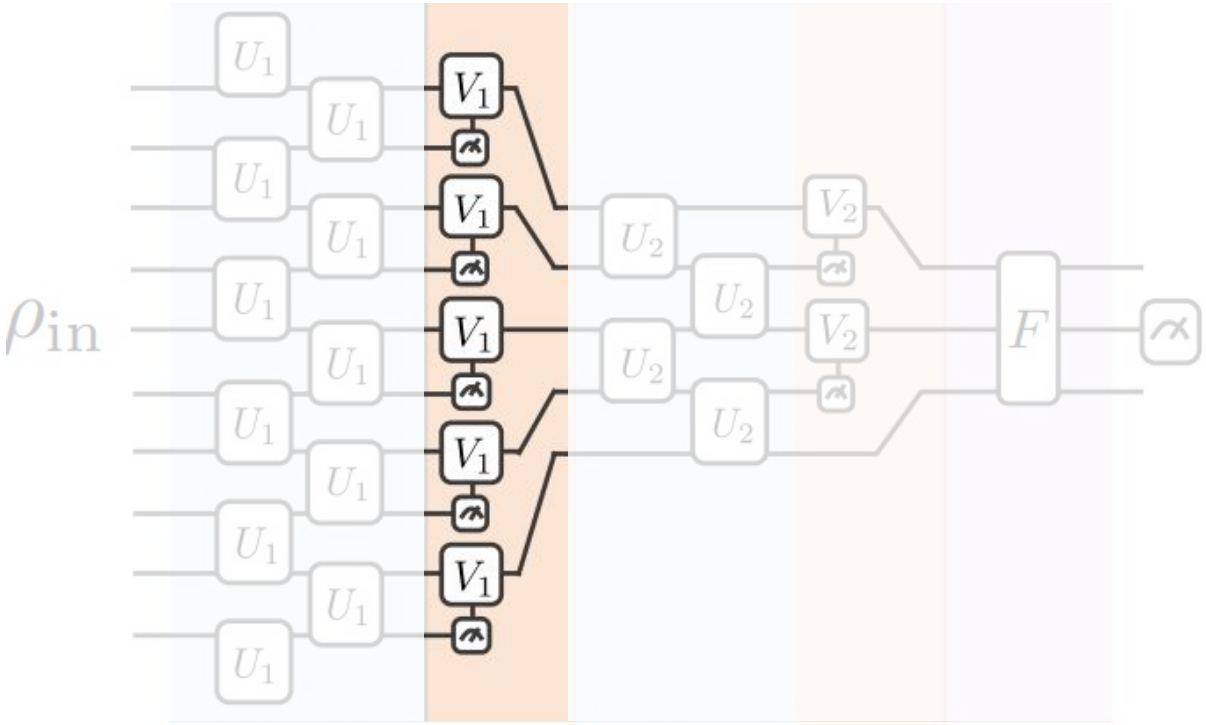

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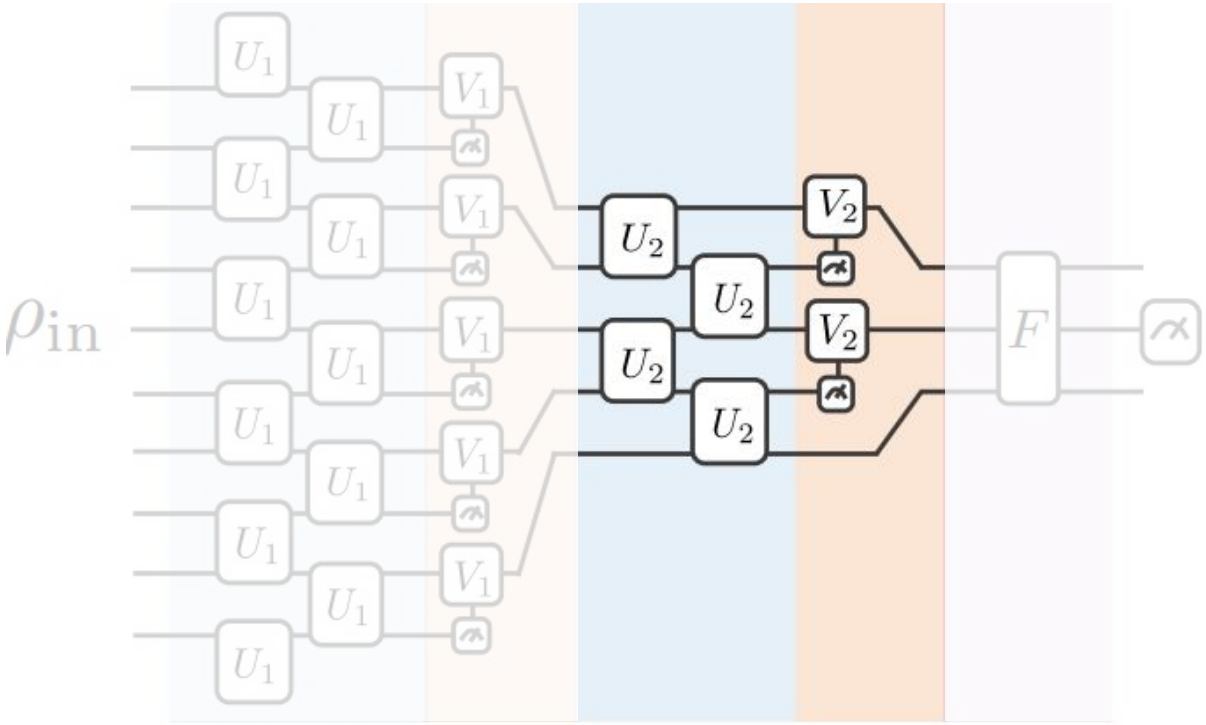

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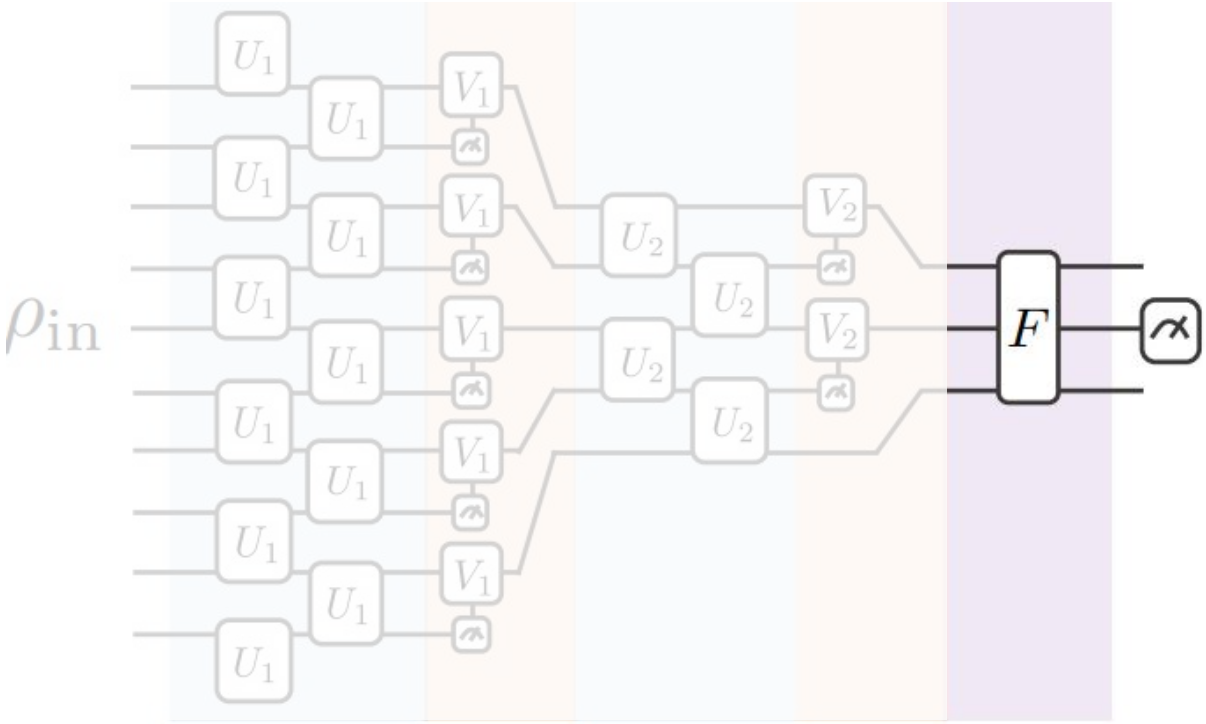
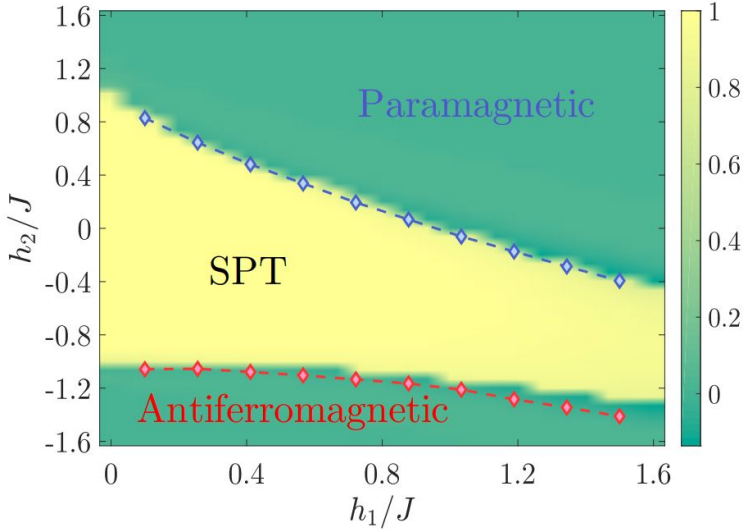


Applications

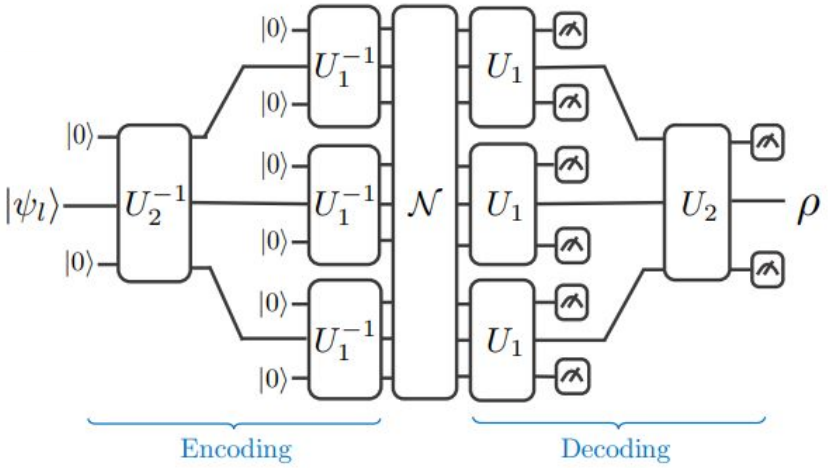
Phase detection

1D Haldane Chain

$$H = -J \sum_{i=1}^{N-2} Z_i X_{i+1} Z_{i+2} - h_1 \sum_{i=1}^{N} X_i - h_2 \sum_{i=1}^{N-1} X_i X_{i+1}$$



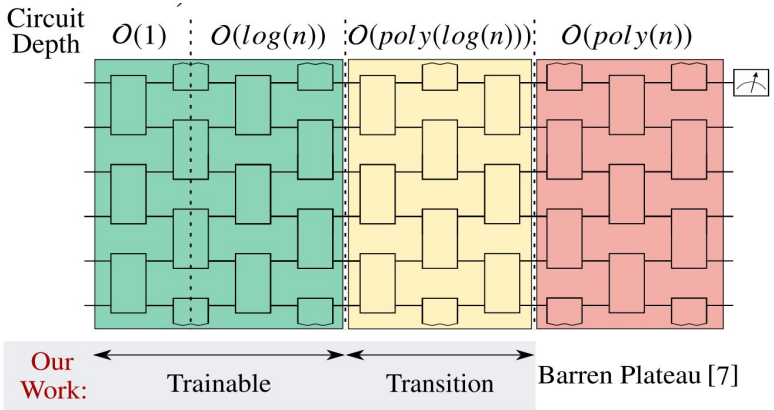
Quantum error correction



Barren plateau for the QCNN: first intuition

Previous work: the hardware-efficient ansatz does **not** have barren plateau if:

1. We have a local-cost function
2. We have a logarithmic number of layers



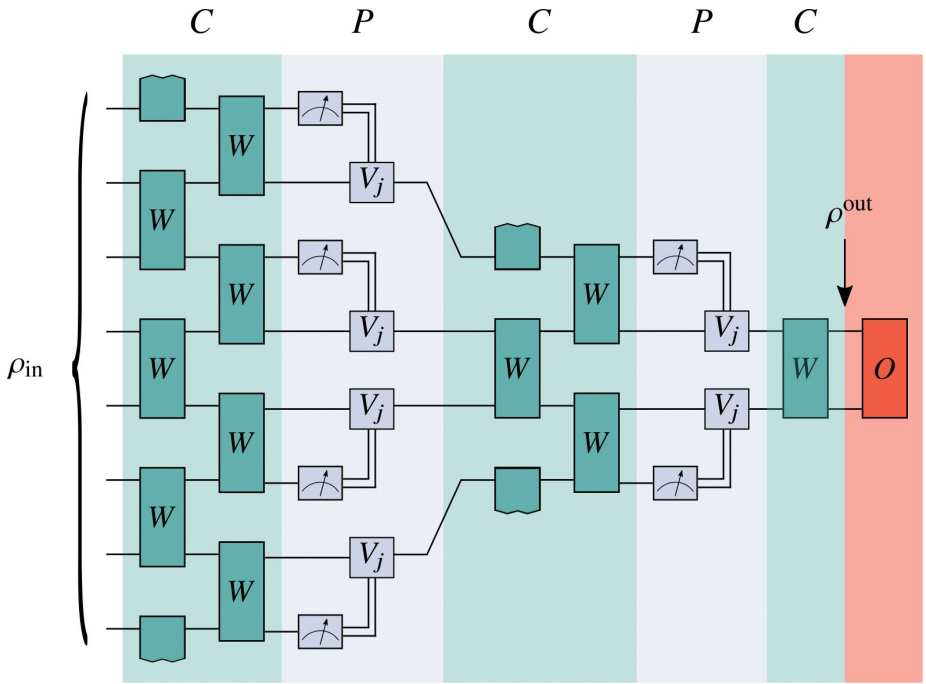
Cerezo et al, *Cost-Function-Dependent Barren Plateaus in Shallow Quantum Neural Networks*, arXiv:2001.00550

Intuition: QCNN won't have any barren plateau because of the logarithmic number of layers

Problem: the QCNN is **not** the hardware-efficient ansatz => we have to do some work

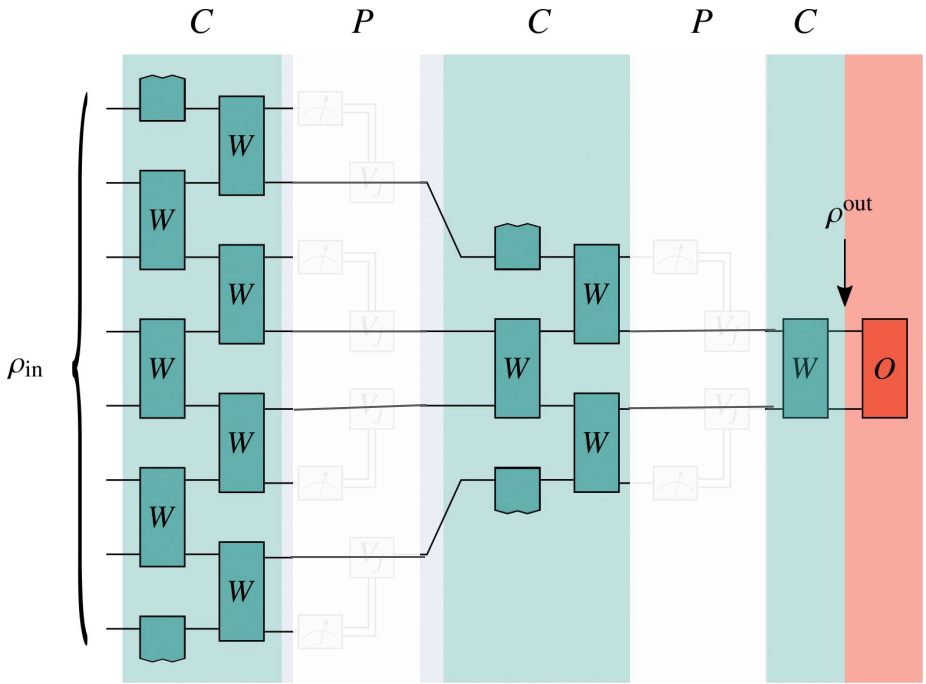
How to study this architecture?

First simplification: modify the circuit to avoid conditional measurement between two blocks



How to study this architecture?

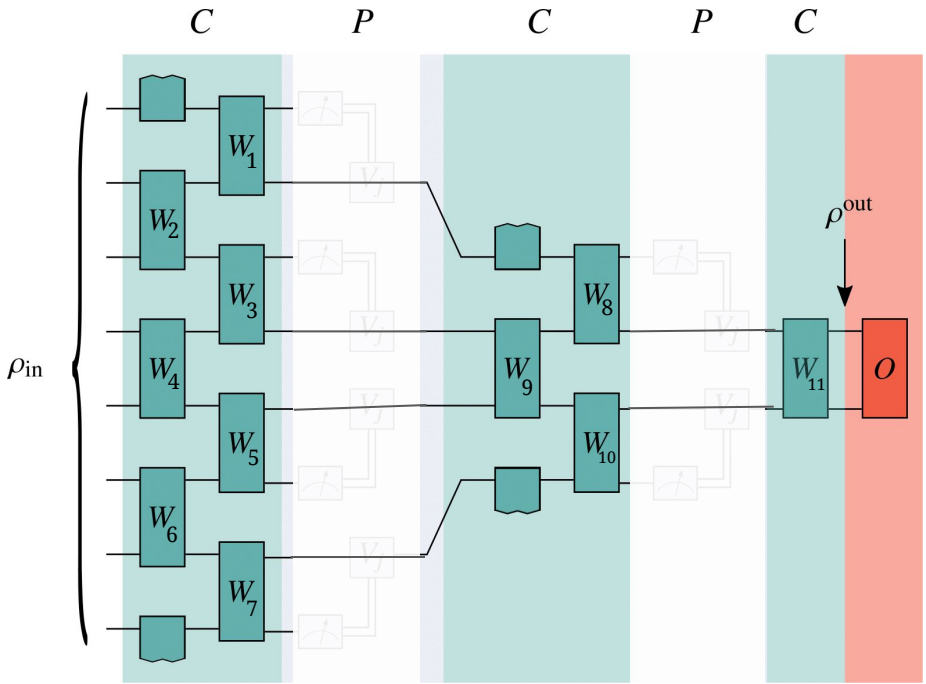
Second simplification: remove the conditional measurements (doesn't change the statistics)



Reason: property of the Haar measure: $\mu_{\text{Haar}}(WV) = \mu_{\text{Haar}}(W)$

How to study this architecture?

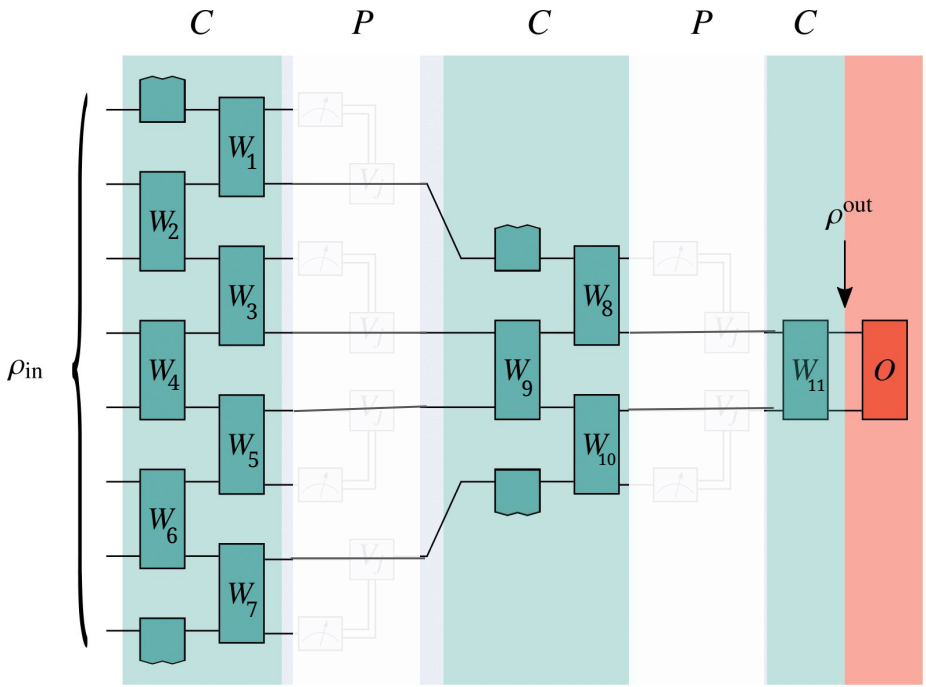
Third simplification: we decorrelate the unitaries



Reason: this should give a lower bound on the correlated version (mix of analytical and empirical evidence)

How to study this architecture?

Fourth simplification: linear cost-function with respect to the input state



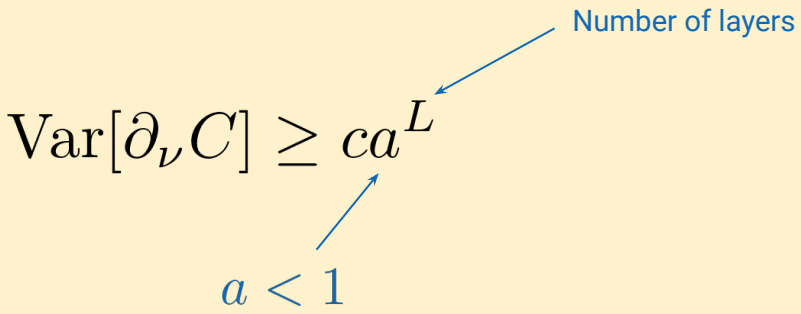
$$C(\theta) = \mathrm{Tr}[OU_\theta \rho_{\text{in}} U_\theta^\dagger]$$

Example: classification (0 vs 1)

$$C(\theta) = \frac{1}{M_0} \sum_{\rho_i \in \mathcal{S}_0} \mathrm{Tr}[U(\theta)\rho_i U^\dagger(\theta) O] - \frac{1}{M_1} \sum_{\rho_i \in \mathcal{S}_1} \mathrm{Tr}[U(\theta)\rho_i U^\dagger(\theta) O]$$

Result

Result: with the hypotheses described above, the variance of the QCNN has at worst an inverse polynomial scaling, i.e. it is trainable



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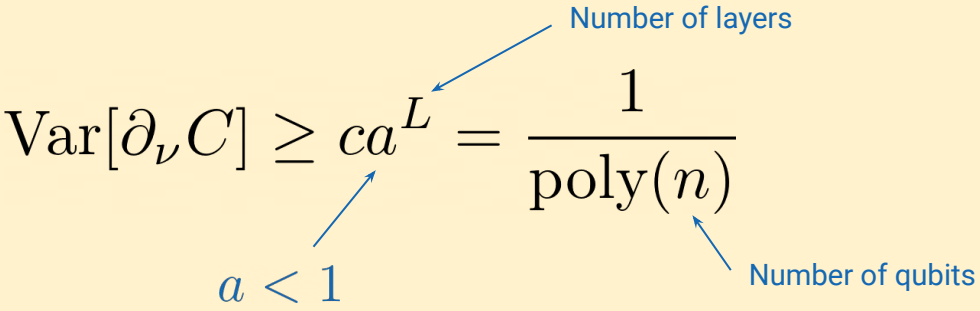
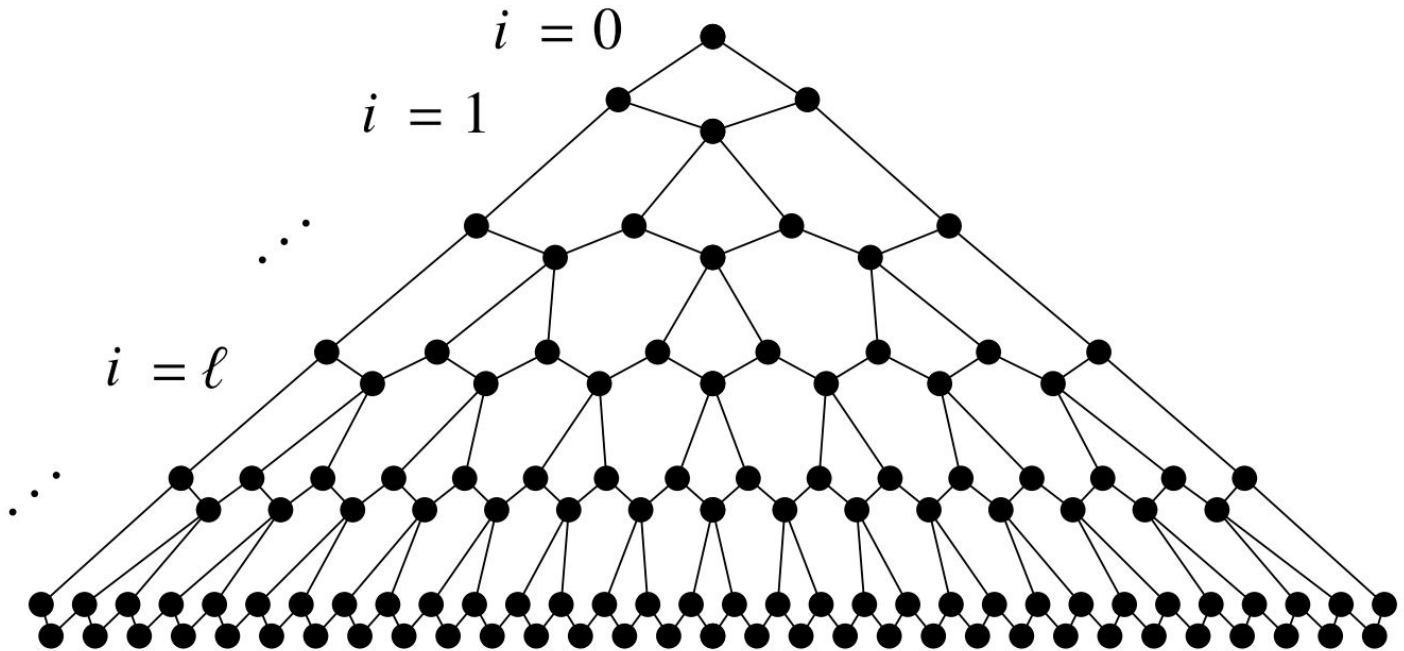


Proof: Graph Recursion Integration Method (GRIM)



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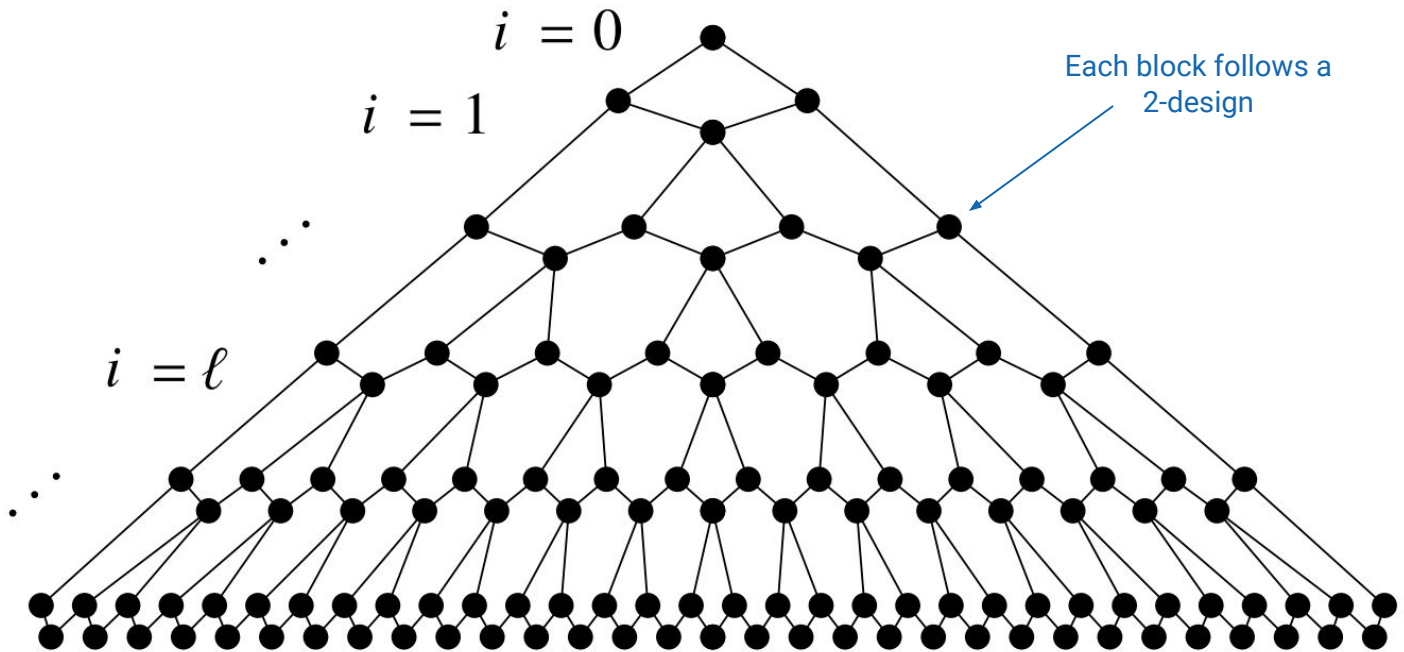

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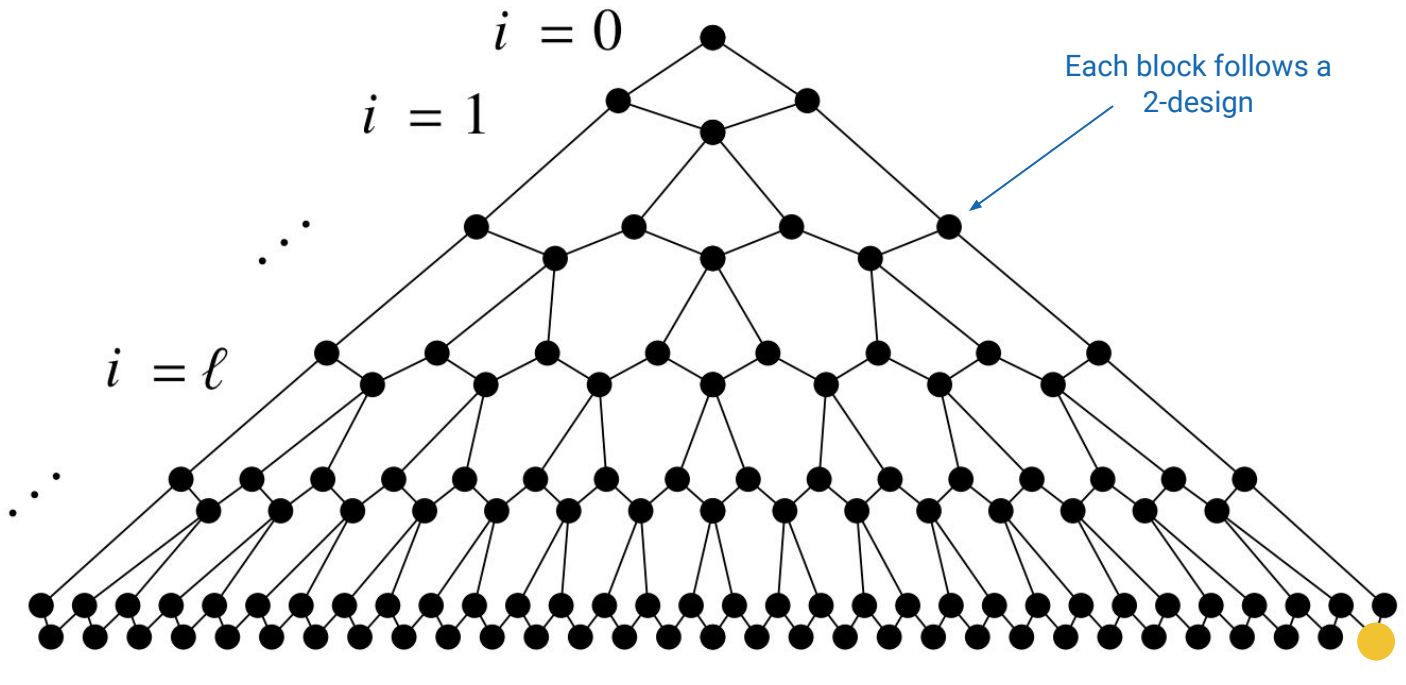
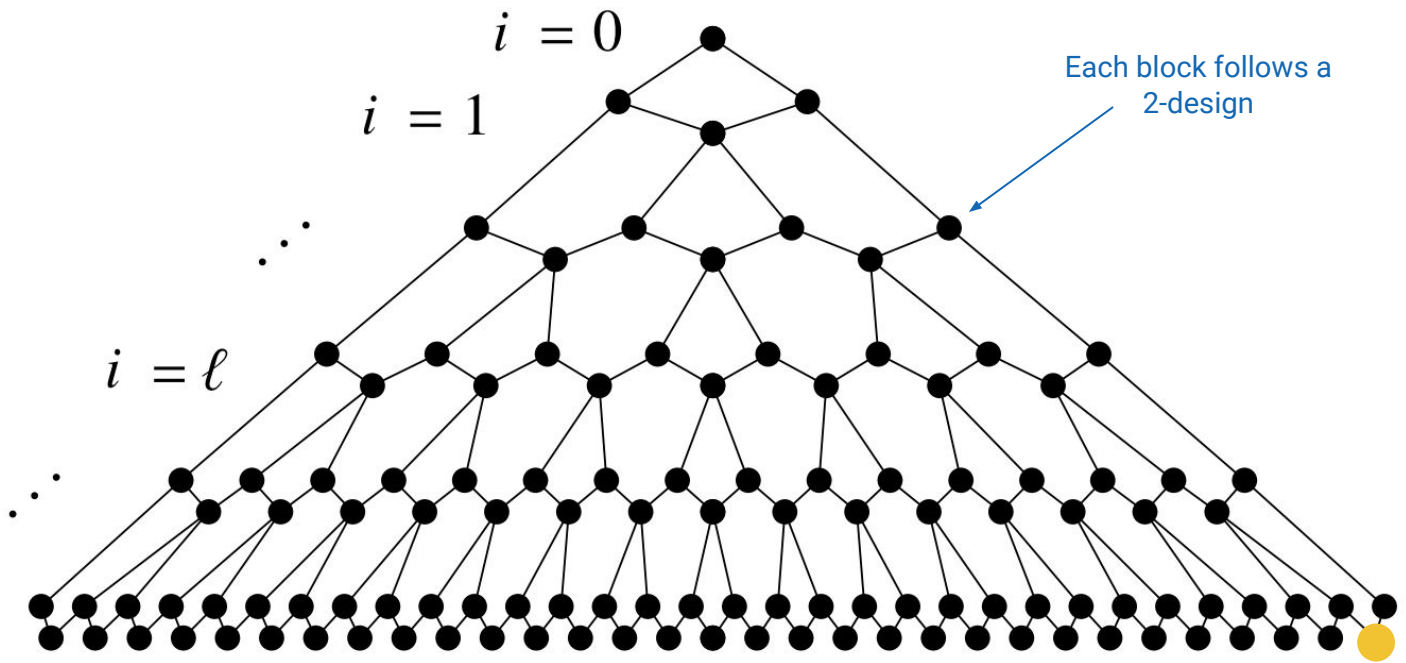


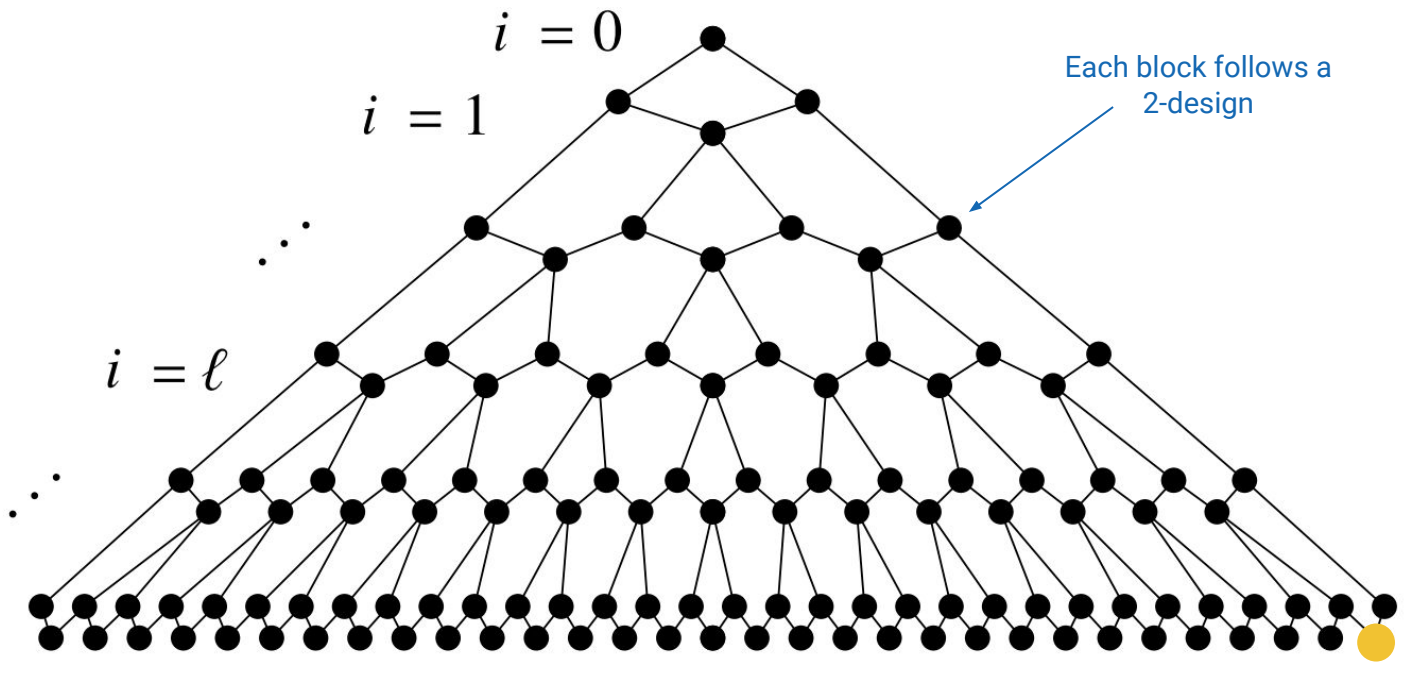
Proof: Graph Recursion Integration Method (GRIM)

Goal: obtain the variance of the gradient $\int_U |\partial_\nu C|^2$



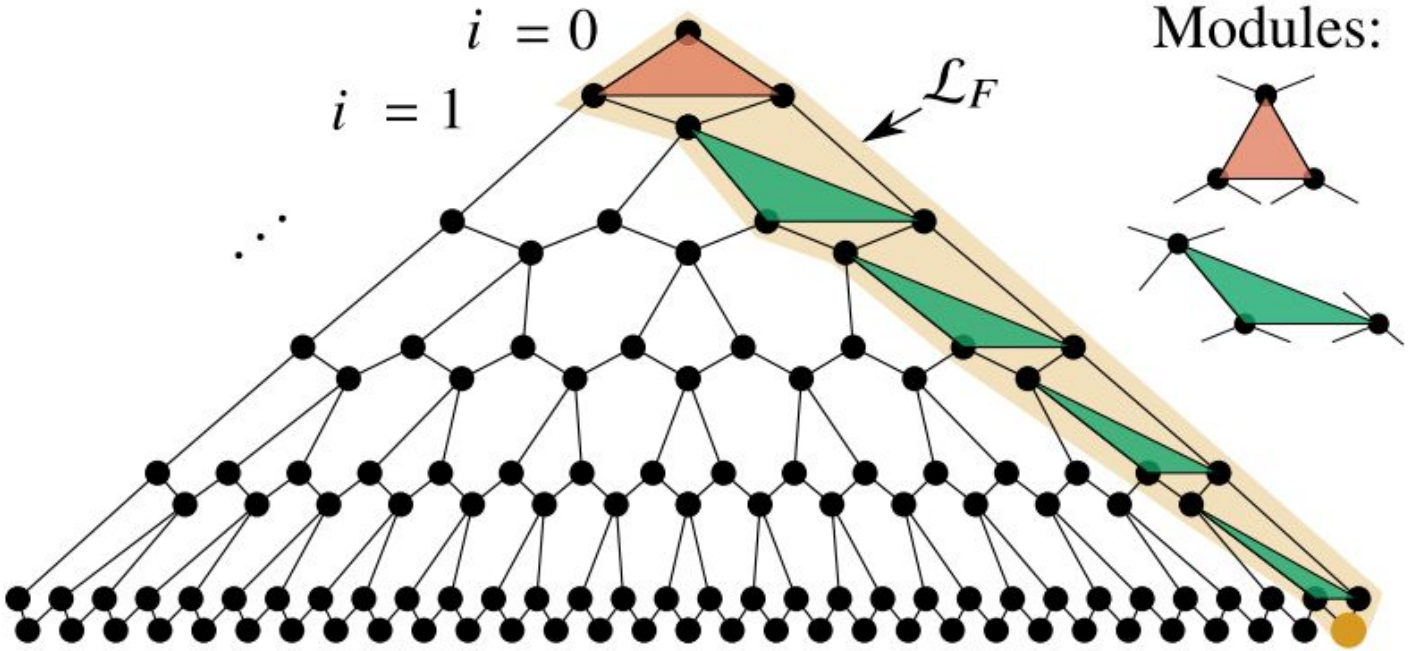
Proof: Graph Recursion Integration Method (GRIM)

Problem: way too many terms to integrate



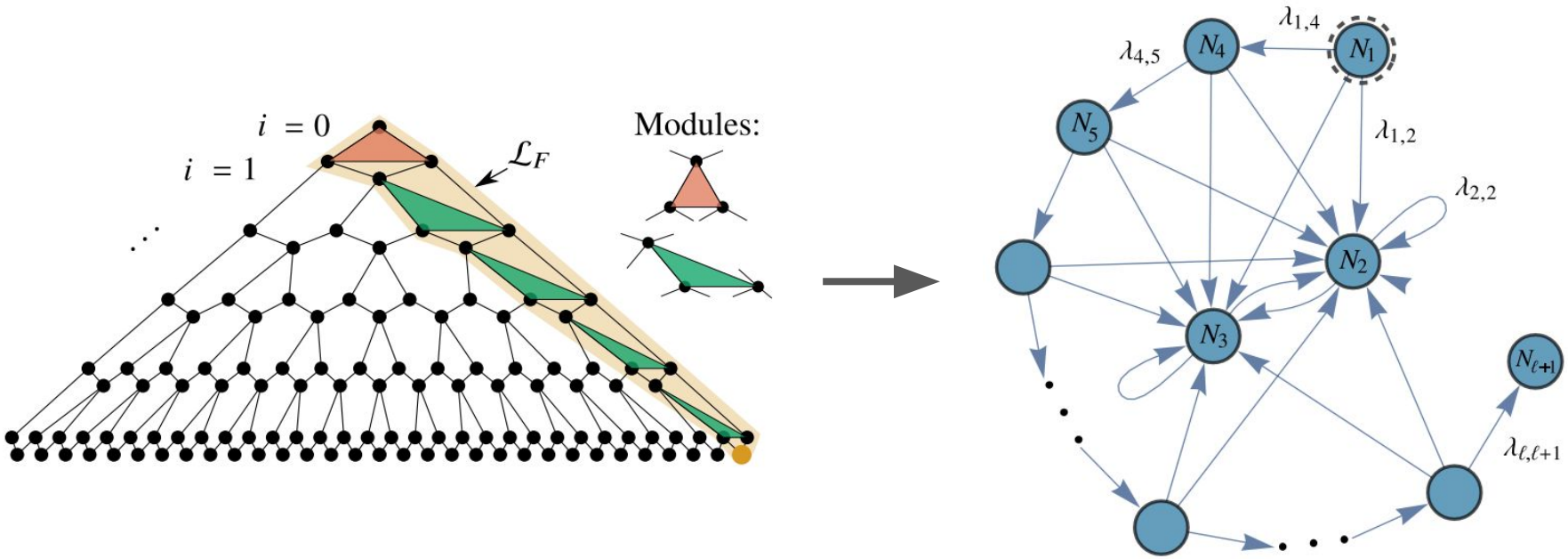
Proof: Graph Recursion Integration Method (GRIM)

Idea: causal cone + module decomposition



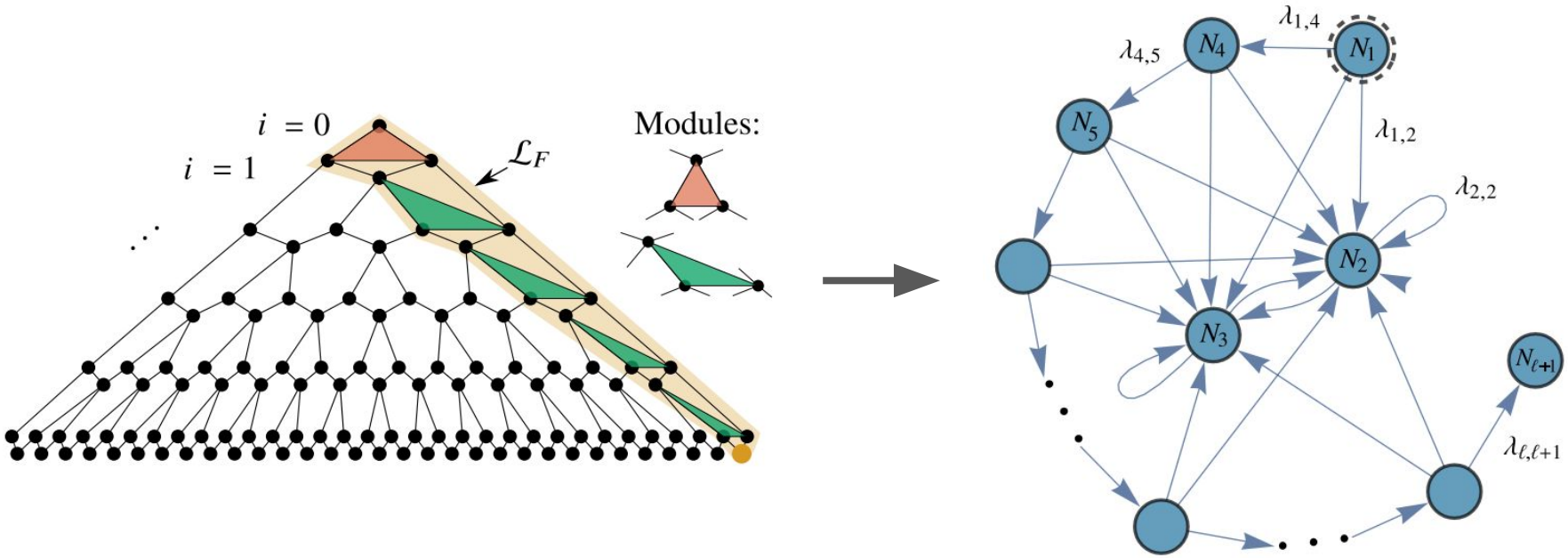
Proof: Graph Recursion Integration Method (GRIM)

Method: keep track of all the integrals in a graph



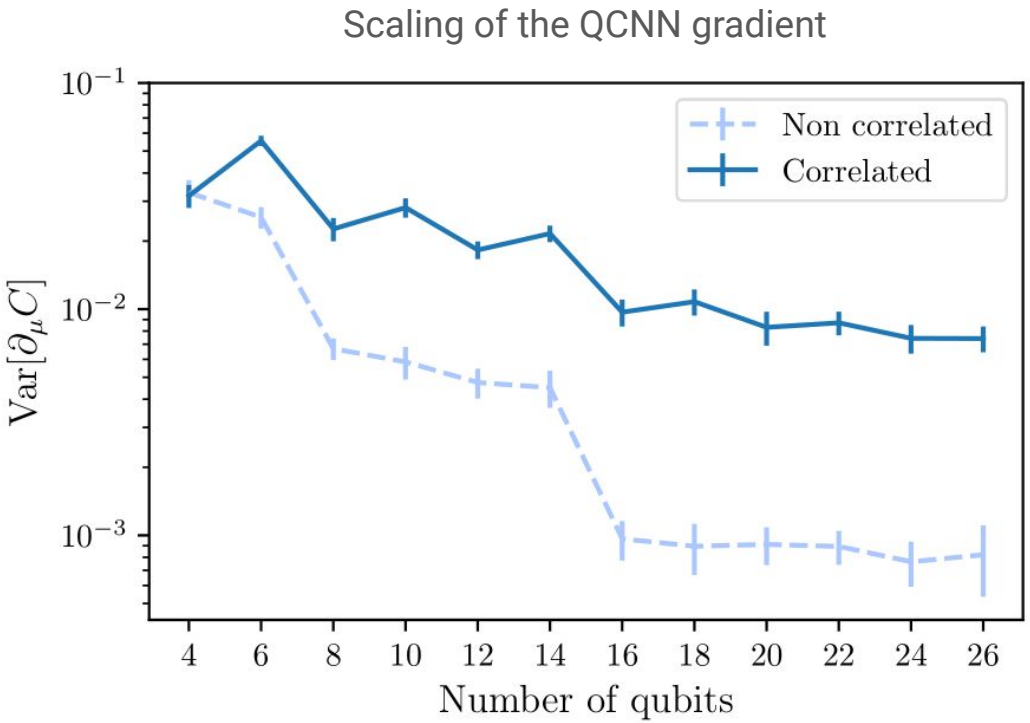
Proof: Graph Recursion Integration Method (GRIM)

Result: variance = sum over all paths of length log(n) in the graph



Numerical experiments

Simulator: Cirq + TensorFlow Quantum



Future work

- How to study correlated architectures systematically?
- How to study non-linear cost-functions? (e.g. mean-squared error)
- Can we deduce the trainability of MERA from our result?
- Can we use GRIM to analyze other architectures
- How useful are QCNNs in practice?

Thanks for your attention